Moral Dilemma

What would happen IF I moved the lever?
Hypothetical Thinking

Conditional Sentences
“I will be discussing a kind of conditional ... typically expressed in English by subjunctive conditionals. Here are some examples: ‘if I were to strike this match there would be an explosion’, ... This kind of counterfactual is intimately connected with laws, explanation, causation, choice, knowledge, memory, measurement, chance, the asymmetry of past and future, etc; a veritable Who’s Who of philosophically and scientifically significant concepts. Philosophers may disagree about the order of explanation among these items and counterfactuals but everyone ought to agree that we would make significant progress understanding them all if we had an account of what makes this kind of counterfactual statement true/false.” — B. Loewer
1 Conditionals between disciplines

1. Conditionals give *concrete form* to *reasoning*.

2. Conditionals exploit *laws* (regularities, generalisations)

   ➞ They are basically everywhere.
1 Conditionals between disciplines

Logic

Philosophy

Mathematics

Linguistics

Conditionals

Cognitive Sciences

Computational Sciences
1 Conditionals between disciplines

Logic

Philosophy

Mathematics

Linguistics

Cognitive Sciences

Computational Sciences
1 Conditionals between disciplines

- Logic
- Philosophy
- Mathematics
- Computational Sciences
- Linguistics
- Cognitive Sciences

Conditionals
Hypothetical Thinking

 Conditional Sentences
Responsibility

• If he hadn’t dropped the bottle, I wouldn’t have slipped.
• His actions caused my slipping.
Hypothetical Thinking

Conditional Sentences ↔ Causality
conditional sentences

and

causal reasoning

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COURSE PLAN

Conditional sentences and causal reasoning

GENERAL INTRODUCTION

Lecture 1: The logic of conditionals: The standard view

Tutorial 1: Challenges for the similarity approach

Lecture 2: Bayes Nets and Causal Bayes Nets

Tutorial 2: Counterfactuals as Interventions

SPECIFIC TOPICS

Seminar 1 (Practicum, part 1): 3 challenges for the framework
(I will first introduce the challenges individually, then you can choose and work in groups on one of them for 40 minutes)

Seminar 2: The relation between the similarity approach and the causal approach

Seminar 3: Using Logic Programming to model causal inferences

Seminar 4 (Practicum, part 2): presentations and discussion
(each group will shortly present their ideas, we will discuss them and I will comment on the state of the arts on each of the challenges)
The logic of conditionals

Plan today

• a bit of history
• the similarity approach ...
• ... and its logic

— BREAK —

• challenges to the similarity approach
• solutions
• and even more challenges
counterfactuals
the standard approach
basic idea

“If I scratched this match, it would light.”
(Goodman)

(1) If this match were struck, it would light, but if this match had been soaked in water overnight and it were struck, it wouldn’t light.
basic idea

“If I scratched this match, it would light.”
(Goodman)

(1) \((\phi \rightarrow \chi) \rightarrow ((\phi \land \psi) \rightarrow \chi)\), i.e. strengthening of the antecedent is not valid.
basic idea

“If I scratched this match, it would light.”
(Goodman)

(1) (Even) if Goethe had survived the year 1832, he would be dead by now.
(2) If Goethe were not dead by now, he would not have survived the year 1832.
basic idea

“If I scratched this match, it would light.”
(Goodman)

(1) \((\phi \rightarrow \chi) \rightarrow (\neg \chi \rightarrow \neg \phi)\), i.e. contraposition is not valid.
(1) If Jones had won the election, Smith would have retired to private life.
(2) If Smith had died last year, Jones had won the election.
(3) If Smith had died last year, Smith would have retired to private life.

“If I scratched this match, it would light.”
(Goodman)
basic idea

“If I scratched this match, it would light.”
(Goodman)

(1) \(((\phi \rightarrow \psi) \land (\psi \rightarrow \chi)) \rightarrow (\phi \rightarrow \chi))\), i.e. transitivity is not valid.
conditionals

If two people are arguing, if $s$, will $q$? Both understand.

They are about to add $s$, hypothetically, to their knowledge and arguing on that basis about $q$. In some cases, if $s$ and $q$ are true, we can say they are fixing their degrees of belief of $s$. If it turns out false, the matter of degree is rendered void. If either party believes certain, the question ceases to mean anything except as a question about what follows certain laws on hypotheses. See below.
conditionals

Ramsey receipt

This is how to evaluate a counterfactual:

• First, add the antecedent hypothetically to your stock of beliefs
• second, make whatever adjustments that are required to maintain consistency (without modifying the hypothetical belief in the antecedent);
• finally, consider whether or not the consequent is then true.
basic idea

“If I scratched this match, it would light.”

(Goodman)
Basic Idea

A sentence

*If it had been the case that* A; it would have been the case that* C

is true in the actual world *w₀* iff *C* is true in all possible worlds in which

(a) *A* is true, and which

(b) in other respects are *maximally similar* to *w₀*. 
Language

- Extend the language of propositional logic with a new binary operator $\sim$

- We will read `$\phi \sim \psi$' as 'If it had been the case that $\phi$, it would have been the case that $\psi$'.
Frames and Models

- We will interpret the languages in **frames** $F = \langle W, \prec \rangle$, where
  - $W \neq \emptyset$
  - $\prec$ is a function which assigns to every $w \in W$ a **strict partial ordering** $\prec_w$ on some subset $W_w$ of $W$. 
Frames and Models

- A **model** is a triple $M = \langle W, \preceq, V \rangle$, where $\langle W, \preceq \rangle$ is a frame and $V$ is a function which assigns a truth value to every atomic sentence in every world.
- $M, w \models \varphi$ says that the formula $\varphi$ is true in $w$ of $M$.
- We write $[\varphi]_M$ to refer to $\{w \in W | M, w \models \varphi\}$, the proposition expressed by $\varphi$ in $M$. 
truth conditions
Add the following clause to the list of truth conditions for the standard connectives.

\[ M, w \models \varphi \rightarrow \psi \iff \]

for every \( u \in W_w \cap [\varphi]_M \) the following holds:

- there is some \( u' \in [\varphi]_M \) such that \( u' \leq_w u \),
- and \( M, u'' \models \psi \) for every \( u'' \in [\varphi]_M \) such that \( u'' \leq_w u' \).
Similarity Approach- formally

The limit assumption
For every $w \in W$, the relation $\prec w$ is well-founded (i.e. every subset of $W_w$ has a minimal element).

Suppose the frame $F=\langle W, \prec \rangle$ satisfies the limit assumption and consider model $M=\langle W, \prec, V \rangle$. The following holds:

$M, w \models \phi \leadsto \psi$ iff $M, u \models \psi$ for every closest $[[\phi]]_M$-world $u$ to $w$. 
The Logic of the similarity approach
(The minimal system P)

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
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<tbody>
<tr>
<td><strong>Taut:</strong></td>
<td>If ( \varphi ) has the form of a classical tautology, then ( \vdash \varphi )</td>
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<tr>
<td><strong>CI:</strong></td>
<td>( \vdash \varphi \rightarrow \varphi )</td>
</tr>
<tr>
<td><strong>CC:</strong></td>
<td>( \vdash ((\varphi \rightarrow \psi) \land (\varphi \rightarrow \chi)) \rightarrow (\varphi \rightarrow (\psi \land \chi)) )</td>
</tr>
<tr>
<td><strong>CW:</strong></td>
<td>( \vdash (\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow (\psi \lor \chi)) )</td>
</tr>
<tr>
<td><strong>ASC:</strong></td>
<td>( \vdash ((\varphi \rightarrow \psi) \land (\varphi \rightarrow \chi)) \rightarrow ((\varphi \land \psi) \rightarrow \chi) )</td>
</tr>
<tr>
<td><strong>AD:</strong></td>
<td>( \vdash ((\varphi \rightarrow \chi) \land (\psi \rightarrow \chi)) \rightarrow ((\varphi \lor \psi) \rightarrow \chi) )</td>
</tr>
<tr>
<td><strong>MP:</strong></td>
<td>( \varphi \rightarrow \psi, \varphi \vdash \psi )</td>
</tr>
<tr>
<td><strong>REA:</strong></td>
<td>If ( \vdash \varphi \leftrightarrow \psi ), then ( \varphi \rightarrow \chi \vdash \psi \rightarrow \chi )</td>
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The Logic of the similarity approach
(The minimal system P)

Strengthening of the antecedent
If in P the scheme (ASC) is strengthened to
\[(AS): (\varphi \leadsto \chi) \rightarrow ((\varphi \land \psi) \leadsto \chi)\]

one gets the system K\leadsto, which is just K in disguise.
Similarity Approach- formally

The Logic of the similarity approach
(The minimal system $P$)

**Weak Centering:** $w \in W_w$ for every $w \in W$, and for no $v \in W_w$ it holds that $v \prec_w w$.

Imposing this constraint means the next rule gets valid.
Modus Ponens for $\sim$ ($MP\sim$):

$\phi \sim \psi, \phi \vdash \psi$
The Logic of the similarity approach
(The minimal system P)

**Almost connectedness:** for any \( u,v \in W_w \), either \( u=v \) or \( u \prec_w v \) or \( v \prec_w u \).

Together with the Limit Assumption this condition brings the Conditional Excluded Middle in its train (CEM):

\[(\varphi \rightarrow \psi) \lor (\varphi \rightarrow \neg \psi)\]
(1) If Bizet and Verdi had been compatriots, Bizet would have been Italian.

(2) If Bizet and Verdi had been compatriots, Verdi would have been French.

(3) If Bizet and Verdi had been compatriots, either Verdi would have been French or Bizet would have been Italian.
The Logic of the similarity approach

What about this one?

\((\text{SDA}): ((\varphi \lor \psi) \sim \chi) \rightarrow (\varphi \sim \chi) \land (\psi \sim \chi)\)

- Is this principle valid in the system P? (Think about it from a semantic point of view.)
- What do you think about the intuitive plausibility of the principle?
(1) If Mary or Sue had been at the party, it would have been a lot more fun.

(2) If Bizet and Verdi had been compatriots, either Verdi would have been French or Bizet would have been Italian.

(3) If Spain had fought with the Axis or the Allies, she would have fought with the Axis.
References

primary texts:


overviews:

• http://paulegre.free.fr/Teaching/ESSLLI_2008/index.htm